# Constrained Initial Guidance Algorithm

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**An algorithm is presented for determining two-body trajectories that originate on a given two-body orbit, intercept a moving target at a free final time, and are generated by an impulsive velocity change of fixed magnitude. By using a coordinate system defined by the initial and final position vectors of the interceptor, the problem is first reduced to one of solving for the real root of a quartic equation that corresponds to minimum flight time between two specified points subject to the velocity change constraint. An iterative procedure then is used to correct the targeted positon until convergence is obtained with respect to the position of the target. Numerical simulation results presented indicate excellent algorithm performance. Typically, less than 20 iterations are needed to reduce miss distance to less than 1 m if perfect target position information is available. The algorithm provides a rapid method of determining the performance boundaries for systems with specified maximum impulse capability.**

## Nomenclature

$a$ = coefficient in equation for $\Delta V_1$
$\hat{a}_i$ = intercept coordinate frame unit vector, $i = 1,2,3$
$b$ = coefficient in equation for $\Delta V_1$
$c$ = distance between $I$ at $t_0$ and $T$ at $t_f$
$\boldsymbol{c}$ = chordal vector, $= \boldsymbol{r}_T - \boldsymbol{r}_S$,
$h$ = orbital areal velocity constant of the interceptor
$I$ = interceptor
$(\hat{\imath},\hat{\jmath},\hat{k})$ = inertial unit vector triad
$p$ = semilatus rectum of the interceptor
$q_i$ = quartic equation coefficients, $i = 1,2,3,4$
$\boldsymbol{r}_I$ = position vector of the interceptor
$\boldsymbol{r}_S$ = position vector of the launcher satellite
$\boldsymbol{r}_T$ = position vector of the target
$S$ = launcher satellite
$T$ = target
$t_{est}$ = estimated transfer time
$t_f$ = actual transfer time
$t_0$ = initial time (taken as zero)
$U_i$ = velocity components of the launcher satellite in the intercept frame, $i = 1,2,3$
$\boldsymbol{V}_I$ = velocity vector of the interceptor after impulsive change
$\boldsymbol{V}_p$ = $V_1\hat{a}_1 + V_2\hat{a}_2$
$\boldsymbol{V}_S$ = velocity vector of the launcher satellite
$\boldsymbol{V}_T$ = velocity vector of the target
$V_1, V_2, V_3$ = radial, transverse, and normal components, respectively, of $\boldsymbol{V}_I$
$\boldsymbol{V}_{1c}, \boldsymbol{V}_{1\rho}$ = component vectors of $\boldsymbol{V}_I$ along skewed axes
$\Delta\boldsymbol{V}$ = impulsive change in the velocity vector of the interceptor
$\Delta V_i$ = component of $\Delta\boldsymbol{V}$, $i = 1,2,3$
$\Delta V_{max}$ = magnitude of the impulsive velocity change
$\Delta\boldsymbol{V}_P$ = $\Delta V_1\hat{a}_1 + \Delta V_2\hat{a}_2$
$\gamma$ = transfer angle
$\mu$ = gravitational parameter
$\rho^2$ = $V_{max}^2 - \Delta V_3^2$

## Introduction

THE problem considered is that of determining the minimum time of flight intercept trajectory for a satellite-launched interceptor with limited velocity change ($\Delta\boldsymbol{V}$) capability. The solution to this problem may be greatly simplified if the launch of the interceptor is considered impulsive and the resulting trajectories are modeled as segments of Keplerian orbits. Short flight times are of primary interest and the resulting trajectories are generally hyperbolic.

Related problems are the classical orbital boundary-value problem of Gauss,[1] and Lambert's[2-4] two-body, two-point, boundary-value problem. The differences in the present problem and the preceding one are that 1) a constraint on the magnitude of the initial velocity impulse is specified a priori, 2) the

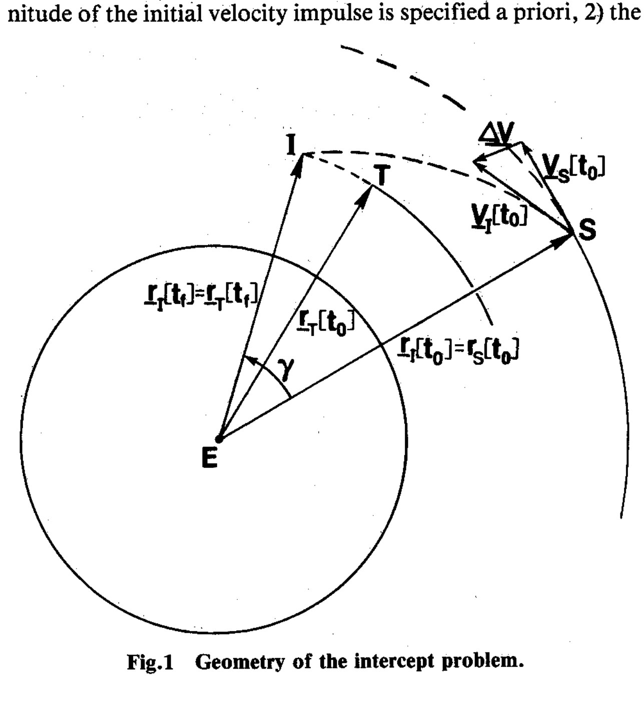


**Fig.1 Geometry of the intercept problem.**

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final velocity is free, and 3) the time of flight is not fixed, but must be such that interception occurs. The desired solution trajectory is the one for which the flight time between the satellite and target is minimized subject to the constraint.

A similar technique for generation intercept trajectories technique is due to Gedeon,[5] who used tabular data. A technique described by Battin[6] and expanded upon by Stuart[7] yields unique trajectories when terminal velocity constraints are considered. However, that technique is not directly applicable to the problem of determining the minimum time of flight intercept trajectory for energy-limited systems. To the authors' knowledge, the algorithm presented here is the first for constrained initial velocity magnitude.

## Solution Approach

The first step in solving the constrained two-body, two-point, boundary-value problem of interest is to write the interceptor's velocity vector in terms of the satellite's orbital velocity vector, plus an unknown change $\Delta V$ in velocity (see Fig. 1). The component of $\Delta V$ perpendicular to the intercept plane is then found. By choosing a particular coordinate system, we obtain one equation in the remaining two unknown components of $\Delta V$. The connection of this equation with a related equation in classical two-body, two-point, boundary-value problems is discussed. The constraint on $|\Delta V|$ is next used to get a quartic equation in terms of one of the impulse components. The real roots of this quartic correspond to *feasible* intercept trajectories.

After determining the appropriate root of the quartic equation and the corresponding trajectory, an iterative procedure is used to adjust the targeted position until convergence to the target's position is achieved. Results are presented of studies of the algorithm's performance.

## Formulation of the Quartic Equation

Formulation of the quartic equation is based on the standard two-body, two-point, boundary-value problem formula relating the initial velocity vector and the initial and subsequent position vectors in a Keplerian orbit (see, for example, Ref. 3, pp. 128–130, 241–244). In the present context, this equation expresses the target position vector $r_T$ at the final time (the interceptor's final position) in terms of the following parameters: The initial interceptor (and satellite) position vector $r_S$, the post impulse velocity of the interceptor $V_I$, the transfer angle $\gamma$, the parameter $p$ of the intercept trajectory, and the gravitational parameter $\mu$. From this equation, we may solve for the initial velocity,

$$V_I = \frac{\sqrt{\mu p}}{r_S r_T \sin\gamma}\left[r_T - r_s + \frac{r_S}{p(1-\cos\gamma)} r_S\right] \tag{1}$$

We can write Eq. (1) in terms of the components of the impulse vector and the initial ($r_S$) and final ($r_T$) positions of the interceptor. These quantities are defined in a dextral, orthogonal, Earth-centered coordinate system containing the unit vector triad $(\hat{\imath},\hat{\jmath},\hat{k})$. Consider the geometry depicted in Fig. 2. A second coordinate system is chosen that has an associated unit vector triad $(\hat{a}_1,\hat{a}_2,\hat{a}_3)$, such that $\hat{a}_1$ is in the direction of the initial interceptor position vector, $\hat{a}_3$ is perpendicular to $r_S$ and $r_T$, and $\hat{a}_2$ follows from the right-hand rule.

Explicitly:

$$\hat{a}_1 = r_S / r_S \tag{2a}$$

$$\hat{a}_2 = \hat{a}_3 \times \hat{a}_1 \tag{2b}$$

$$\hat{a}_3 = (r_S \times r_T)/|r_S \times r_T| \tag{2c}$$

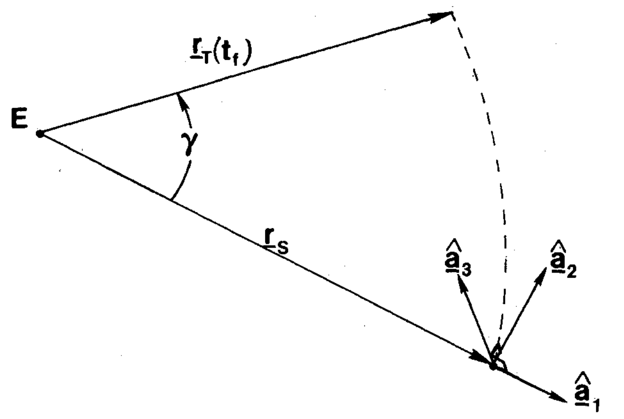


**Fig. 2 Definition of unit vectors $\hat{a}_j$, $j = 1,2,3$.**

Using these unit vectors, we may write the interceptor velocity vector in terms of the satellite velocity plus the impulse vector

$$V_I = (U_1 + \Delta V_1)\hat{a}_1 + (U_2 + \Delta V_2)\hat{a}_2 + (U_3 + \Delta V_3)\hat{a}_3 \tag{3a}$$

where

$$U_i = V_S \cdot \hat{a}_i, \qquad i = 1,2,3 \tag{3b}$$

$$\Delta V_i = \Delta V \cdot \hat{a}_i, \qquad i = 1,2,3 \tag{3c}$$

Next, we write the parameter in terms of the areal velocity constant $h$ (commonly, specific or massless angular momentum) as

$$p = h^2/\mu \tag{4}$$

Since the $\hat{a}_2$ component of the interceptor velocity is, by construction, perpendicular to the radius vector and lies in the intercept plane, we can write $h$ as

$$h = r_S(U_2 + \Delta V_2) \tag{5}$$

and then rewrite Eq. (1) in the form

$$V_I = \left(\frac{U_2 + \Delta V_2}{r_T \sin\gamma}\right) \times \left\{ r_T - \left[1 - \left[\frac{r_T \mu}{r_S^2 (U_2 + \Delta V_2)^2}\right](1-\cos\gamma)\right] r_S \right\} \tag{6}$$

By substituting $V_I$ from Eq. (3a) for the interceptor's postimpulse velocity into Eq. 6, we obtain an equation that may be separated into planar (in the intercept plane) and nonplanar components of Eq. (6). The nonplanar component is obtained by dotting the resulting equation with $\hat{a}_3$ to obtain

$$U_3 + \Delta V_3 = 0 \tag{7}$$

One planar component is obtained by taking the dot product of $V_I$ and $\hat{a}_1$. The result is

$$U_1 + \Delta V_1 = (U_2 + \Delta V_2) \times \left\{ \frac{\cos\gamma}{\sin\gamma} - \frac{r_S/r_T}{\sin\gamma} + \left(\frac{\mu}{r_S \sin\gamma}\right)\left[\frac{1-\cos\gamma}{(U_2+\Delta V_2)^2}\right] \right\} \tag{8}$$

(The third dot product yields $V_2 \equiv V_2$.)

For convenience, let us solve Eq. (8) for $\Delta V_1$ and write the result as

$$\Delta V_1 = a(U_2 + \Delta V_2) + b/(U_2 + \Delta V_2) - U_1 \tag{9a}$$

where $a$ and $b$ are given by

$$a = \cos\gamma/\sin\gamma - (r_S/r_T)/\sin\gamma \tag{9b}$$

$$b = \mu(1-\cos\gamma)/(r_S \sin\gamma) \tag{9c}$$

The geometric significance of Eq. (9a) is illustrated in Fig. 3, which shows a pair of curves in the $\Delta V_1 \Delta V_2$ plane that have the general appearance of hyperbolas.

As $\Delta V_2 \rightarrow -U_2$, the curves approach the vertical asymptote. Also, as $V_2 \rightarrow \pm\infty$, the curves approach asymptotically a straight line of slope $a$ and $\Delta V_1$-axis intercept $aU_2 - U_1$. The example curves shown in Fig. 3 were obtained using $\gamma = 30$ deg, $r_S/r_T = 8/7$, $r_S = 4$, $\mu = 1$ (canonical units), $U_1 = 0$ (circular launcher orbit), and $U_2 = 0.5$. For this data, $a \simeq -0.55366$ and $b \simeq 0.067$.

As one might expect, the curves defined by Eqs. (9) are the same as those described by the terminus of the velocity vector $V_1$ expressed as the sum of the chordal velocity

$$V_{1c} = (V_2/r_T)\csc\gamma c \tag{10a}$$

and the radial velocity

$$V_{1\rho} = \left[\mu(1-\cos\gamma)/(V_2 r_S^2 \sin\gamma)\right] r_S \tag{10b}$$

(see Ref. 3, pp. 242-245, especially problem 6-4).

The vertical asymptote in Fig. 3 is in the radial direction and the other is in the chordal direction. Note that $V_{1\rho}\cdot\hat{a}_1 \neq V_1$, but $V_{1c}\cdot\hat{a}_2 = V_2$. Since $r\cdot c = r_S(r_T\cos\gamma - r_S)$, by dotting the sum of $V_{1c}$ and $V_{1\rho}$ with $a_1$, we recover Eq. (9a).

In the intercept plane, the constraint on $|\Delta V|$ is a circle [see Eq. (12)] if we use normal, radial, and transverse components of $\Delta V$. If we transform the constraint equation into skewed coordinates, it is an ellipse. We have chosen the first approach.

The impulse magnitude constraint can be written as

$$\Delta V_{max}^2 = \Delta V_1^2 + \Delta V_2^2 + \Delta V_3^2 \tag{11}$$

Because $\Delta V_3$ can be found immediately from Eq. (7), we define a circle in the intercept plane with radius $\rho$, which represents the constraint on $\Delta V_1$ and $\Delta V_2$. Analytically, we have

$$\rho^2 = \Delta V_{max}^2 - \Delta V_3^2 = \Delta V_1^2 + \Delta V_2^2 \tag{12}$$

Since solutions for $\Delta V_1$ and $\Delta V_2$ must satisfy both Eq. (9a) and Eq. (12), at most four possible solutions for $\Delta V_1$ and $\Delta V_2$ can be obtained. Geometrically, these points are intersections of Eq. (9a) curves with the circle defined by Eq. (12) (see Fig. 3).

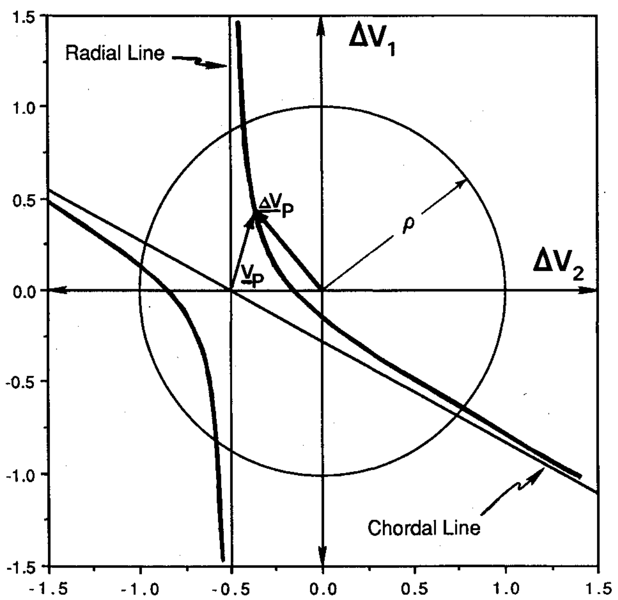


**Fig. 3 Geometric interpretation of the equations for $\Delta V_1$ and $\Delta V_2$.**

Equation (9a) may be used to find $\Delta V_1^2$. Then, by substituting that expression into Eq. (12), we get

$$\rho^2 - \Delta V_2^2 = \left[a(U_2 + \Delta V_2) + [b/(U_2 + \Delta V_2)] - U_1\right]^2 \tag{13}$$

which may be rewritten in the form of a quartic equation,

$$q_4\Delta V_2^4 + q_3\Delta V_2^3 + q_2\Delta V_2^2 + q_1\Delta V_2 + q_0 = 0 \tag{14}$$

with the coefficients defined as follows:

$$q_4 = a^2 + 1 \tag{15a}$$

$$q_3 = 4a^2U_2 + 2U_2 - 2aU_1 \tag{15b}$$

$$q_2 = 6a^2U_2^2 + U_1^2 - 6aU_1U_2 + U_2^2 + 2ab - \rho^2 \tag{15c}$$

$$q_1 = 4a^2U_2^3 - 6aU_1U_2^2 - 2bU_1 + 2U_1^2U_2 + 4abU_2 - 2U_2\rho^2 \tag{15d}$$

$$q_0 = a^2U_2^4 - 2aU_1U_2^3 - 2bU_1U_2 + U_1^2U_2^2 + b^2 - U_2^2\rho^2 + 2abU_2^2 \tag{15e}$$

All of the roots of the quartic equation represent mathematically possible solutions for $\Delta V_2$, but, as indicated previously, only real roots are admissible solutions. The maximum real root generally is the one that yields the minimum flight time between $S$ and $T$ when $r_S > r_T$ and the launcher orbit is circular. This can be explained with the help of Fig. 3. For the largest root, $\Delta V_{2_{max}}$, $\Delta V_1$ is negative. The combination of a negative $\Delta V_1$ and a positive $\Delta V_2$ (in the direction of the launcher velocity) will result in a high-energy trajectory that has its periapse point below the launch point for small $\gamma$. Hence, the interceptor's distance from the Earth center will monotonically decrease, resulting in what is probably the shortest flight time. For other geometries, there may be minimum-time trajectories in which the interceptor first travels outward. The negative roots correspond to trajectories produced by slowing the projectile down; for large magnitude negative $\Delta V_2$, retrograde trajectories are produced.

To insure that the minimum time is found, the times corresponding to all real roots should be checked.

## Application to Targeting

Although the $\Delta V$ found by the preceding method provides a trajectory that connects the points $S$ and $T$ in space, it does not necessarily result in a trajectory that reaches the position $r_T$ at the time the target reaches that position. The following procedure may be used along with solutions for $\Delta V$ to obtain an intercept trajectory.

1) Given the states of the target and the satellite at $t_0 = 0$, estimate $t_{est}$ and the necessary transfer time, and predict the target position at $t_{est}$. If $\Delta V_{max}$ is large, a good first estimate for $t_{est}$ is $c/\Delta V_{max}$, where $c$ is the line-of-sight, or chordal, distance between the target and satellite at the initial time $t_0$.

2) Set up the intercept coordinate system defined by Eqs. (2). Determine the transfer angle via the cross-product rule, and the satellite's initial velocity components in the intercept frame from Eq. (3b).

3) Determine the solution for $\Delta V_3$ using Eq. (7).

4) Form the quartic equation (14) using Eqs. (9), (11), and (15).

5) Determine the real roots of the quartic equation. The maximum real root is generally the minimum-time solution for $\Delta V_2$.

6) Determine the $\Delta V_1$ component from Eqs. (9).

7) Form the interceptor's velocity vector from Eq. (3a) and determine the orbital elements of the intercept trajectory.

8) Calculate the actual transfer time $t_f$ using Lambert's time-of-flight equation (see, for example, Ref. 3, Chap. 7) and compare $t_f$ with the estimated transfer time used in step 1. If the absolute value of the difference in these times is greater than a preset tolerance (0.0001 s was found to be adequate for the example cases), then adjust the estimated transfer time using

$$t_{est} = t_{est} + (t_f - t_{est})/2 \tag{16}$$

Predict the target position at the new $t_{est}$ and go to step 2. Note that for smaller values of $\Delta V_{max}$ a better first guess for $t_{est}$ is $c/\rho$.

## Numerical Results

A computer code was developed to solve the constrained initial guidance problem using the preceding algorithm. A fourth-order Runge-Kutta routine was included in the code to numerically integrate the equations of motion from launch to closest approach of the interceptor to the target to study algorithm accuracy and investigate the effects of convergence criteria on algorithm performance. Although the motion is completely defined analytically, it was more convenient to use numerical integration.

The overall figure of merit for algorithm accuracy was taken to be the uncertainty distance, defined as the miss distance (the minimum distance between the interceptor and the target) when exact initial states and the true target state information are used. This uncertainty distance arises solely from numerical imprecision and the use of a finite number of iterations to find $\mathbf{\Delta V}$. For the perturbed case, the miss distance is of particular interest. By applying the algorithm, the expected miss distance can be determined easily for known errors in the $\mathbf{\Delta V}$ impulse vector. The overall algorithm performance is determined by the number of iterations required to satisfy a suitable convergence criteria, say, uncertainty distance less than 1 m.

Five different scenarios were selected to demonstrate the performance of the initial guidance algorithm. The five sets of orbital elements for the satellite and target are shown for each scenario in Table 1. For each scenario, a $\Delta V_{max}$ value of 10 km/s was first used. The value was increased to 12 km/s to see how changes in $\Delta V_{max}$ would affect performance of the algorithm. The quartic coefficients and roots are given in Table 2 for each case.

**Table 1 Orbital elements for the example cases**

| | | $\Omega$, deg | $\omega+f$, deg | $i$, deg | $e$ | $a$, km |
|---|---|---|---|---|---|---|
| Case 1 | Satellite | 30 | 30 | 30 | 0 | 6878 |
| | Target | 50 | 20 | 35 | 0 | 6578 |
| Case 2 | Satellite | 30 | 10 | 30 | 0 | 6878 |
| | Target | 50 | 20 | 60 | 0 | 6578 |
| Case 3 | Satellite | 30 | 10 | 30 | 0 | 6878 |
| | Target | 50 | 20 | 89 | 0 | 6578 |
| Case 4 | Satellite | 15 | 0 | 30 | 0 | 6878 |
| | Target | 20 | 0 | 30 | 0.2 | 7078 |
| Case 5 | Satellite | 15 | 0 | 30 | 0 | 6878 |
| | Target | 20 | 5 | 30 | 0.2 | 7078 |

**Table 2 Coefficients of the quartic equations ($\mathbf{\Delta V}$ = 10 km/s)**

| | $q_4$ | $q_3$ | $q_2$ | $q_1$ | $q_0$ |
|---|---|---|---|---|---|
| Case 1 | 1.093 | 16.944 | −18.609 | −1265.106 | −4688.005 |
| Case 2 | 1.214 | 20.766 | 4.611 | −1358.558 | −5005.529 |
| Case 3 | 1.186 | 14.437 | −29.130 | −814.612 | −1851.555 |
| Case 4 | 1.660 | 35.027 | 168.039 | −594.573 | −4295.120 |
| Case 5 | 1.451 | 28.850 | 94.269 | −1017.144 | −5143.543 |

**Table 3 Algorithm performance and accuracy data**

| | | UD,[a] m | $c$, km | Time, s | $N$ |
|---|---|---|---|---|---|
| Case 1 | 10 km/s $\Delta V$ | 0.4720 | 1948.0 | 118.4549 | 4 |
| | 12 km/s $\Delta V$ | 0.2470 | 1821.0 | 99.4084 | 5 |
| Case 2 | 10 km/s $\Delta V$ | 0.5318 | 4968.2 | 302.2469 | 9 |
| | 12 km/s $\Delta V$ | 0.2099 | 4567.1 | 245.2375 | 7 |
| Case 3 | 10 km/s $\Delta V$ | 0.5515 | 4561.3 | 338.5360 | 19 |
| | 12 km/s $\Delta V$ | 0.5801 | 3815.0 | 234.5621 | 11 |
| Case 4 | 10 km/s $\Delta V$ | 0.2922 | 2365.3 | 154.2675 | 7 |
| | 12 km/s $\Delta V$ | 0.2348 | 2121.0 | 124.8258 | 4 |
| Case 5 | 10 km/s $\Delta V$ | 0.4230 | 3236.1 | 200.8854 | 10 |
| | 12 km/s $\Delta V$ | 0.3465 | 2878.6 | 160.5100 | 8 |

[a]UD = uncertainty distance.

**Table 4 Required $\mathbf{\Delta V}$ components in intercept coordinates**

| | Components, km/s | $\Delta V_1$, km/s | $\Delta V_2$, km/s | $\Delta V_3$, km/s |
|---|---|---|---|---|
| Case 1 | 10 km/s $\Delta V$ | −4.2901 | 8.6372 | 2.6446 |
| | 12 km/s $\Delta V$ | −4.9447 | 10.5300 | 2.9443 |
| Case 2 | 10 km/s $\Delta V$ | −5.5613 | 7.9946 | 2.2711 |
| | 12 km/s $\Delta V$ | −6.2876 | 10.0171 | 2.0308 |
| Case 3 | 10 km/s $\Delta V$ | −3.8267 | 7.4224 | 5.5013 |
| | 12 km/s $\Delta V$ | −4.7281 | 9.7708 | 5.1163 |
| Case 4 | 10 km/s $\Delta V$ | −8.9186 | 4.4149 | 0.9838 |
| | 12 km/s $\Delta V$ | −10.7726 | 5.1574 | 1.1625 |
| Case 5 | 10 km/s $\Delta V$ | −8.0181 | 5.9415 | 0.6394 |
| | 12 km/s $\Delta V$ | −9.5976 | 7.1648 | 0.7433 |

**Table 5 Inertial impulse vectors and intercept point**

| | | Inertial Component | | |
|---|---|---|---|---|
| | | $\hat{i}$ | $\hat{j}$ | $\hat{k}$ |
| Case 1 | VTBG[a] | −0.4608 | 1.7754 | −2.7093 |
| | Position | 1792.3436 | 6075.6470 | 1773.1605 |
| Case 2 | VTBG | −7.1978 | −0.9018 | 6.8832 |
| | Position | 1579.3947 | 5204.9114 | 3699.2552 |
| Case 3 | VTBG | −1.3498 | −5.5995 | 8.1784 |
| | Position | 3031.2297 | 3734.3345 | 4487.3678 |
| Case 4 | VTBG | −9.7676 | 1.9965 | 0.7795 |
| | Position | 4766.5125 | 3030.3125 | 702.8195 |
| Case 5 | VTBG | −9.2069 | 3.3813 | 1.9490 |
| | Position | 4271.9157 | 3661.8633 | 1143.1215 |

[a]VTBG = velocity to be gained.

Table 3 gives the uncertainty distance as well as the chordal distance, transfer time, and the number of iterations required for convergence. A tolerance of $1 \times 10^{-4}$ s in the transfer time was determined to be adequate for obtaining uncertainty distances of less than 1 m for all cases considered. As can be seen from Table 3, the algorithm performs very well for the selected convergence criteria; the uncertainty distances for all cases are $<1$ m. As one might expect, the uncertainty distance is the greatest for case 3, for which the largest out-of-plane component of $\mathbf{\Delta V}$ is required.

The number of iterations required to achieve convergence varies greatly for the five cases. This is due to the geometrical differences of the five orbit scenarios. The largest number of iterations, 19, is required for case 3, which requires the largest $\Delta V_3$. It is also interesting to note that, with the exception of case 1, a greater $\Delta V_{max}$ capability tends to reduce the number of required iterations. The $\Delta V_j$, $j = 1,2,3$, required for the intercept, are shown in Table 4. Computational speed is under 1 CPU second on a Harris 800 minicomputer. The required impulse vector, or velocity-to-be-gained, and the intercept

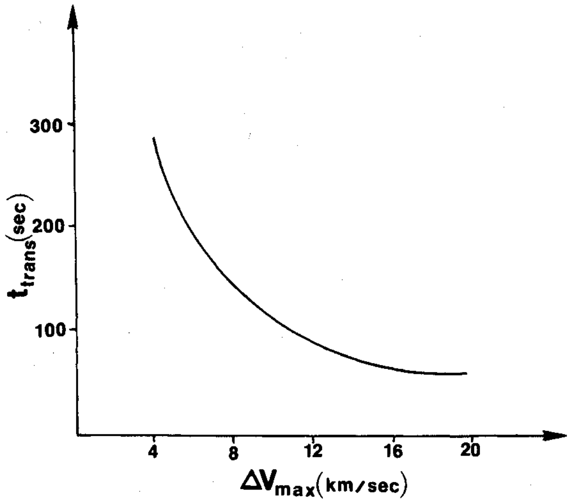

Fig. 4 Transfer time vs $\Delta V_{max}$.

point are given in Table 5 for each of the five scenarios and $\Delta V_{max} = 10$ km/s.

All of the cases considered here represent posigrade intercepts with respect to the intercept coordinate system. The required inertial impulse vector $\Delta V$ and the intercept position are given in Table 5 for each of the five scenarios and $\Delta V_{max} = 10$ km/s.

### Available Energy vs Transfer Time

It is also interesting to examine how changes in available energy, represented by $\Delta V_{max}$, affect the minimum transfer time for final initial conditions. A plot was generated to illustrate the relationship of $\Delta V_{max}$ and transfer time for case 1 (see Fig. 4). Information of this type is needed to determine the $\Delta V$ required for particular intercept geometries and transfer times.

## Conclusions

An initial guidance algorithm for exoatmospheric interceptors has been derived under the assumptions that the interceptor's trajectory is Keplerian after an impulsive change in velocity that is subject to a magnitude constraint. It is reasonable to use a constraint like this for solid rocket boosted interceptors and projectiles launched from electromagnetic launchers.

The example cases considered demonstrate that the algorithm is accurate and relatively fast. The two assumptions of Keplerian trajectories and impulsive velocity changes are not unduly restrictive since for a short flight time the interceptor will likely have a short, high-acceleration boost phase. Thus, for scenarios of the type considered in this paper, the algorithm is appropriate for initial guidance of an energy-limited system. For lower acceleration interceptors, results based on the algorithm would provide good first estimates for application of, say, gradient-search type optimization techniques.[8]

The algorithm should be useful as a quick method for evaluating exoatmostpheric interceptors. Useful information, such as regions of reachable space and expected miss distance at intercept may be rapidly computed to establish performace boundaries. Trajectories obtained from the algorithm can be used as nominal trajectories in the evaluation of midcourse and terminal guidance schemes. Although the target trajectories considered in the examples are Keplerian, application of the algorithm requires only that a target trajectory be specified. Hence, the algorithm can also be used if the target is thrusting.

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